

应用 TVD 格式分离求解不可压 N - S 流动

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【摘要】 提出一种新的将连续方程与动量方程分离求解的 TVD 引用方法。对动量方程中的对流项采用三阶精度、迎风 TVD 格式离散, 粘性项和压力项采用中心差分格式离散。压力采用压力替代法求解。计算所用的网格是具有交错网格主要特点的修正非交错网格, 并采用一种压力修正法和 G - S 中心对称迭代法加速收敛。给出了二维管流和二维空穴流动在不同状态下的计算结果, 并与其它解法的计算结果或精确解进行比较。结果表明, TVD 格式不仅能用于不可压流数值计算, 而且还能得到比非 TVD 差分法精度更高的结果。

关键词: TVD 差分 N - S 方程 修正非交错网格 不可压流

1 控制方程

控制方程是 N - S 方程组。采用 Shih 的压力替代法^[1], 不可压流体的 N - S 方程组在任意座标系下的守恒形式:

$$\text{grad} \bar{V} = 0 \quad (1)$$

$$\partial \bar{q} / \partial \tau + \partial \bar{q}_1 / \partial \xi + \partial \bar{q}_2 / \partial \eta = \partial \bar{p}_1 / \partial \xi + \partial \bar{p}_2 / \partial \eta + \partial \bar{g}_1 / \partial \xi + \partial \bar{g}_2 / \partial \eta + F \quad (2)$$

式中: $\bar{q} = \bar{q}/J$, $\bar{f}_1 = (\zeta/J) \bar{f}_1 + (\zeta/J) \bar{f}_2$, $\bar{f}_2 = (\eta/J) \bar{f}_1 + (\eta/J) \bar{f}_2$, $\bar{p}_1 = (\zeta/J) \bar{p}_1 + (\zeta/J) \bar{p}_2$, $\bar{p}_2 = (\eta/J) \bar{p}_1 + (\eta/J) \bar{p}_2$, $\bar{g}_1 = (\zeta/J) \bar{g}_1 + (\zeta/J) \bar{g}_2$, $\bar{g}_2 = (\eta/J) \bar{g}_1 + (\eta/J) \bar{g}_2$, $J = \partial(\zeta \eta) / \partial(x, y)$, $\bar{V} = u \bar{i}_x + v \bar{i}_y$, $\bar{i}_x$ 、 $\bar{i}_y$ 分别是沿 x 、 y 方向的单位矢量。

$$\bar{q} = \begin{bmatrix} u \\ v \end{bmatrix}, \quad \bar{f}_1 = \begin{bmatrix} u^2 \\ uv \end{bmatrix}, \quad \bar{f}_2 = \begin{bmatrix} uv \\ v^2 \end{bmatrix}, \quad \bar{p}_1 = \begin{bmatrix} -p \\ 0 \end{bmatrix}, \quad \bar{p}_2 = \begin{bmatrix} 0 \\ -p \end{bmatrix}$$

$$\bar{F} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}, \quad \bar{g}_1 = \frac{1}{Re} \begin{bmatrix} \mu(\partial u / \partial x) \\ \mu(\partial v / \partial x) \end{bmatrix}, \quad \bar{g}_2 = \frac{1}{Re} \begin{bmatrix} \mu(\partial u / \partial y) \\ \mu(\partial v / \partial y) \end{bmatrix}$$

式中, p 、 u 、 v 、 Re 和 μ 是无量纲量的, 分别代表压力、 x 、 y 方向的速度、雷诺数和粘性系数。

2 TVD 差分法结构及其离散、收敛加速

2.1 TVD 差分法结构

TVD 差分法于 1983 年首先由 Harten 提出^[2], 至今已发展为多种格式^[3]。文献 [4] 给出了一种应用 TVD 求解不可压 N - S 方程的方法。本文采用 Chakravarthy 的 5 点 TVD 差分格式, 其公式详见文献 [3]。其中 $\phi = 1/3$, 该格式具有三阶精度。

2 2 动量与连续方程的离散

方程组 (2) 的粘性项和压力项采用中心差分格式, 而对流项内结点采用三阶精度 TVD 差分格式。(2) 式的 Jacobian 矩阵及右、左特征矢量分别为

$$A = \frac{\partial \bar{q}}{\partial \bar{q}} = \begin{bmatrix} \bar{u} + uK_x & uK_y \\ vK_x & \bar{u} + vK_y \end{bmatrix}, \quad \begin{bmatrix} (u+v)K_y & u(K_y - K_x) \\ -(u+v)K_x & v(K_y - K_x) \end{bmatrix}$$

$$\begin{bmatrix} v/[(u+v)(uK_x + vK_y)] & -u/[(u+v)(uK_x + vK_y)] \\ -K_x/[(K_x - K_y)(uK_x + vK_y)] & -K_y/[(K_x - K_y)(uK_x + vK_y)] \end{bmatrix}$$

这些特征矢量对应特征值 $\bar{u}$ 和 $2\bar{u}$ 。其中 $\bar{u} = uK_x + vK_y$, 沿着 ζ 和 η 方向分别为 $K_x = \zeta/J$, $K_y = \zeta/J$; $K_x = \eta/J$, $K_y = \eta/J$ 。而 $J = \partial(\zeta, \eta)/\partial(x, y)$ 。

将控制方程写成 (1) 的理由为: 在不可压流计算中, 动量方程的离散格式不能探测出不合理的压力场而可能出现棋盘状压力场。为了避免这一问题出现, Harlow 和 Welsh^[5]则提出了交错网格方法。Shih 提出了一种具有交错网格特征的非交错网格方法^[1]。前者的网格, 将速度 u 、 v 和压力 p 分别存储于 3 套不同的网格中, 如图 1a; 后者将 u 、 v 和 p 存储于同一网格的不同计算点上, 如图 1b。其实, 这两种网格系统的基本思想是一致的。

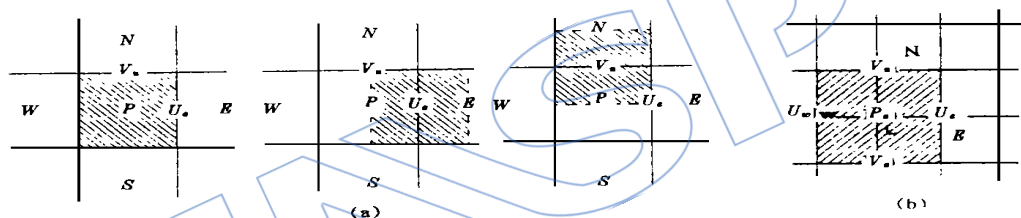


图 1 修正网格 (a) 与一般网格 (b) 对比

可见, 直接从控制方程求的同一控制体的 u 、 v 、 p 并非同一点。在 TVD 差分中, 由于用通量分裂法, 则需要同一点上的 u 、 v 、 p 值。即建立在非交错网格上的, 而通常的这种网格会导致棋盘状压力场。如果采用 Chorin 的拟压力方程, 并采用 TVD 差分来求解不可压 $N-S$ 方程, 那么离散后进行计算则需要大量差值运算。尤其是 u 、 v 控制体中心的压力的插值计算会引起很大的误差。本文作者通过做这方面的工作^[6], 对 u 、 v 控制容积中心点上压力 p 的插值, 曾采用两点式, 也采用四点格式, 但是均未获得成功。写成 (1) 式使得整个离散格式既能适用于交错网格和 Shih 的修正非交错网格, 又能大大减少压力插值计算, 提高求解稳定性和精度。同时, 它还能使我们采用传统的分离式解法。先由沿 x 方向动量方程求出全场 u , 再由沿 y 方向动量方程求出全场 v , 然后由压力替代法求解出压力 p 。

在本文中采用了 Shih^[1]的压力替代法。连续方程由下面压力方程代替

$$p_c^{(n+1)} = p_c^{(n)} - \gamma_p \text{grad} \bar{V} \quad (3)$$

γ_p 的定义见文献 [1], 只是空间和流体速度的一个函数, 而不象 Chorin^[7]的方法所导出的显示格式, 时间显示地出现在格式中。

2 3 收敛加速

方程 (1) 的差分代数方程严格主对角占优^[6], 采用 G-S 点迭代, 并让计算从计算域的左边向右边迭代, 从下边向上边迭代, 完成一轮迭代后, 在下一轮迭代中按反方向迭代。这样

整个边界的影响都传到计算域内而迭代次数只有两次, 从而, 加快了收敛速度。

在本文中, 采用文献 [1] 的压力修正法。其中心思想是采用流线上的动量方程来修正流线上的平均压力。修正量与从动量方程算得的流速与从质量平衡所得的流速之差成正比。

3 结 果

应用本文方法计算如图 (2a) 的管流, 网格为 44×64 , Re 分别为 30、100、500、1000 和 2000。这里体积力忽略。图 3 表明计算结果与精确解吻合很好, 而表 1 给出了 $Re = 1000$ 管流

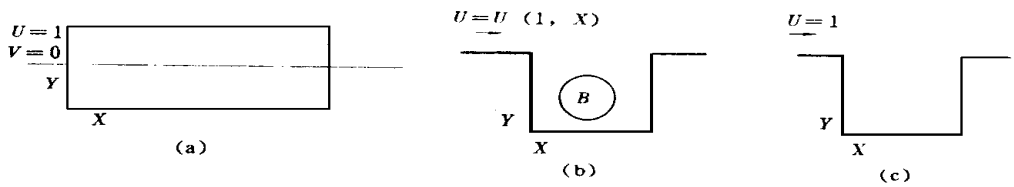


图 2 管流和空穴流

充分发展段的速度值, 此表表明, 在壁点所得的解有较大的相对误差。但总的来看, TVD 格式比非 TVD 格式的精度高。

表 1 充分发展管流 TVD 解与非 TVD 解及精确解比较

y 座标	TVD 44×64		NON - TVD 44×64		精确解
	结果	相对误差%	结果	相对误差%	
0.0078	0.0234	0.48	0.0234	0.48	0.0235
0.0954	0.2725	< 0.01	0.2727	0.07	0.2725
0.2021	0.5452	0.02	0.5454	0.07	0.5450
0.3041	0.7738	0.03	0.7740	0.05	0.7736
0.3919	0.9456	0.03	0.9457	0.04	0.9453
0.4908	1.1114	0.03	1.1113	0.02	1.1111
0.6841	1.3502	< 0.01	1.3501	0.02	1.3503
0.8198	1.4510	0.02	1.4508	0.03	1.4513
0.9197	1.4900	0.02	1.4897	0.04	1.4903
0.9727	1.4986	0.02	1.4983	0.04	1.4989
总误差		0.07		0.09	

本文计算了如图 (2b) 的 $Re = 1$ 并含体积力的空穴流动问题 $F_x = 0, F_y = B(x, y, Re)$, 其中

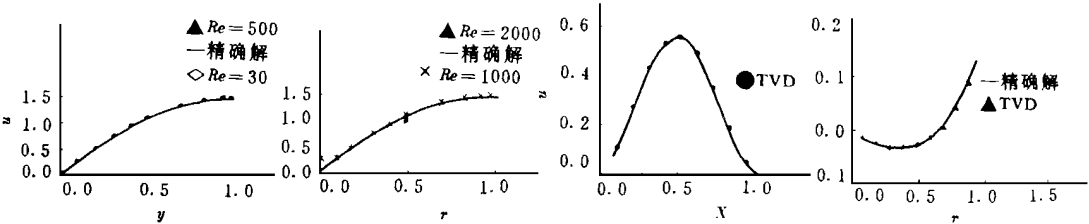


图 3 管流 TVD 解与精确解比较

图 4 空穴流 TVD 解与精确解的比较

函数 $B(x, y, Re)$ 选自文献 [8]。速度 u 和 v 的边界条件为 $Dirichlet$ 条件: 除上边界 $u(x, 1) = 16(x^4 - 2x^3 + x^2)$ 外其他边界均为零。图 4 表明所得解分别在 $x = 0.1$ 和 $y = 0.9$ 处与精确解符合

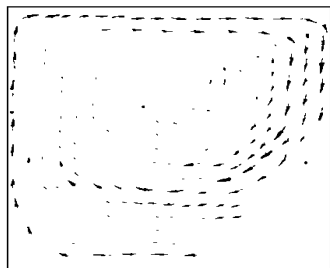


图 5 空穴流速度矢量分布

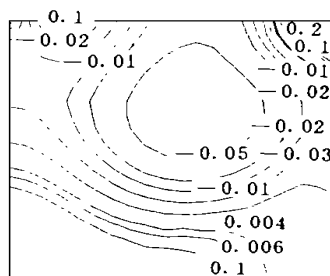


图 6 空穴流压力等值线分布

很好。还计算了如图 (2c) 的空穴流动, 网格为 40×40 , $Re = 100, 400, 500, 1000, 5000$ 。这里忽略了体积力。速度 u 和 v 的边界条件是: 除上边界 $u(x, 1) = 1$ 外其余均为零。图 5 和图 6 分别给出了空穴流动在 $Re = 500$ 状态的速度分布与压力分布。

4 结 论

总变差减小 (TVD) 差分法被成功地用于不可压 $N-S$ 流计算。当采用 $Shih$ 的修正非交错网格, 连续方程与动量方程分离求解, 动量方程的对流项采用三阶精度 TVD 差分格式离散, 可得到高精度解。压力修正方法和 $G-S$ 中心对称迭代法能加速收敛。

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tained from extrapolation. The resulted algebraic equations are tridiagonal block matrix system and solved by sweeping in each direction. So the scheme is a 5 point implicit AF scheme of first-order temporal and third order or second-order spacial accuracy depending on the two TVD factors. Two examples are presented, the first verifies the calculation of third-order scheme with a low speed wind tunnel experiment, the second compares the third-order scheme with the second-order one. The examples indicate that the code results in numerical solutions agree with the experiment, and the third-order scheme has little difference from the second-order one in the solutions, but it decreases the convergent residual greatly. Also it is showed that the two schemes do not agree with each other very well in the throat of the cascade where the flow velocity is equal to the local sound speed.

A METHOD OF USING TVD SCHEME TO CALCULATE INCOMPRESSIBLE FLOWS SEPARATELY

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ABSTRACT The Total Variation Diminishing (TVD) schemes have been adapted to incompressible flow computations. Considering the characters of the computation in the incompressible flow, a new method is given that the TVD scheme is used with separating the continuity equation from momentum equations. The continuum governing equations are discretized on a modified nonstaggered grid. In the momentum equations, the convective terms are discretized by a third-order TVD scheme, and the others are done by central finite difference scheme. The pressures are solved by pressure substitution. Furthermore, a pressure correction procedure and the Gauss-Seidel centrally symmetric iteration algorithm are used to enhance the convergence rate. As a result, the code containing those methods has been applied to computation of two typical flow problems: the channel flow problem and the lid-driven cavity flow problem. A comparison of the present results with the literature data is also given in good agreement. It is shown that the TVD scheme is not only applicable to incompressible flow computation, but also it yields higher accuracy than the non-TVD scheme.

A NEW COMPREHENSION ON STATOR CASCADE UNDER ROTATING STALL

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ABSTRACT Onset points of rotating stall, time-average performance and flow fields after stall appearing, as well as the unstable behaviors for a single-stage axial-flow compressor and an isolated rotor have been measured successfully with hot-film probes and a three-hole cylindrical pressure probe containing fast response transducers. On the basis of many experimental data, the influences of stator cascade in the single-stage compressor on the whole time-average characteristics and the cessation behavior of rotating stall, as well as the stalled flow field have been elaborated in detail. When a single-stage compressor enters the unstable condition, the roles of the stator cascade, such as postponing the appearance of rotating stall, as well as modifying the stalled flow field and cessation form, have been analysed. There is an obvious difference of the unstable behavior between the single-stage axial-flow compressor and the isolated rotor. At high rotor speed ($n^+ > 0.85$), a nonrecoverable stall phenomenon appears for a single stage compressor, but it does not appear for an isolated rotor at any speed.

HEAT TRANSFER FROM SHROUDED ROTATING DISK WITH CENTRIC AIR INFLOW

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ABSTRACT An air-cooled turbine disk was investigated as the model of a rotating